Integrated Scheduling for Gasoline Blending Considering Storage Tanks and Pipe Network

Satoshi Hoshino, Noriyoshi Furuya, and Hiroya Seki

Abstract— An off-site system in a petroleum refining plant mainly consists of storage tanks, gasoline component blenders, and a pipe network that connects tanks with blenders. Therefore, in order to produce a required gasoline on time, these equipments have to be selected and used appropriately at the right time. However, previous studies have not taken into account the pipe network. In this paper, a novel scheduling model with the MILP method for gasoline blending in consideration of storage tanks and a pipe network is proposed. In order to model and formulate the pipe network and constraints, a previously-proposed scheduling model is extended with the use of the set theory. In this regard, two modeling techniques are introduced to reduce a computational cost. Through a case study of the gasoline-blending scheduling, the effectiveness of the proposed and integrated scheduling model is finally shown.

I. INTRODUCTION

Fig. 1 shows a schema of a petroleum refining plant. Arrived oil is, first off, stored into crude oil tanks. After that, gasoline components are refined through an on-site system with refining equipments. A gasoline component, through pipes in an off-site system, is transferred and stored to a component tank or to a product stock tank if the component is already an on-spec product. A gasoline component in a tank is mixed with other components through a blender which consists of a pipe, pump, and line mixer; then, transferred and stored to a product stock tank. Finally, a product gasoline is lifted from a tank for a delivery order.

In this paper, the off-site system which consists of the component tanks, product stock tanks, and blenders for the storage and mixture and the pipe network that connects these equipments for the component or product transfer is the objective. Here, gasoline blending is defined as follows: a process which contains a gasoline production, such as components transfer, mixture, and products storage and a product gasoline delivery. In terms of an efficient off-site system management, we focus on a short-term planning of gasoline blending.

The aim of this paper is to select and use the appropriate storage tanks, blenders, and pipes at the right time to produce and deliver a product gasoline on time for an order. For this purpose, we propose a novel scheduling model in consideration of the pipes in addition to the tanks and blenders. This paper consists of the following sections: related works are described in II to make clear the problems of them and

S. Hoshino and H. Seki are with the Chemical Resources Laboratory, Tokyo Institute of Technology, Yokohama, Kanagawa 226-8503, JAPAN {hosino, hseki}@pse.res.titech.ac.jp

N. Furuya is with the Chemical Engineering Department, Tokyo Institute of Technology, Yokohama, Kanagawa 226-8503, JAPAN furuya@pse.res.titech.ac.jp
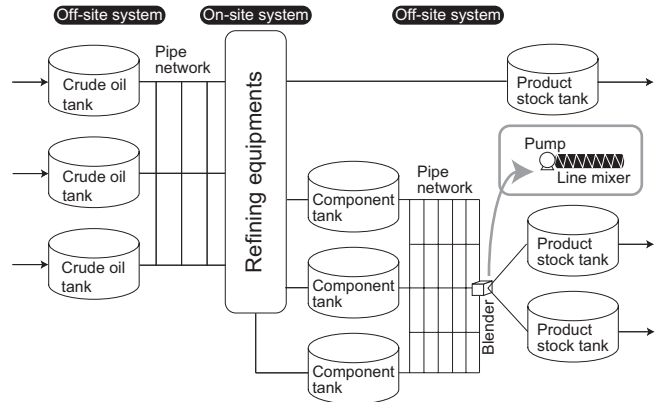

Fig. 1. Schematic View of a Petroleum Refining Plant

the originality of this paper; in III, the scheduling problem for gasoline blending considering storage tanks and pipes is described; the integrated scheduling model is proposed in IV; the effectiveness of the proposed scheduling model is discussed in V; and finally this paper is concluded in VI.

II. RELATED WORKS

Related works, so far, have proposed several scheduling models with the use of mathematical programming methods [1] [2] and meta-heuristic methods [3] [4] [5] for a batch process. In this paper, we adopt a mathematical programming method from the viewpoint of an optimality and computational time (cost) for the scheduling.

Previous works with the use of a mathematical programming method have formulated the scheduling as the Mixed Integer Linear Programming (MILP) problem by representing constraints regarding equipments (if it is assignable) using binary variables. Then, these works have proposed (a) discrete time-based [1], (b) time-slot-based [6], and (c) event-based [7] [8] [9] scheduling models. Glismann *et al.* have, by applying model (a) to the gasoline-blending scheduling, developed an integrated approach to coordinate short-term planning of multi-product blending facilities [10]. Djukanovic *et al.* have formulated a fuzzy linear programming problem to obtain a reasonable non-fuzzy solution [11]. Méndez *et al.* have optimized the off-line blending and short-term planning problems by applying model (b) [12]. Jia *et al.* have developed an efficient MILP formulation based on continuous representation of the time domain, (c) [13].

These previous works, however, have not taken into account the existence of the pipe network. Hence, the pipe scheduling has been conducted hierarchically using heuristics
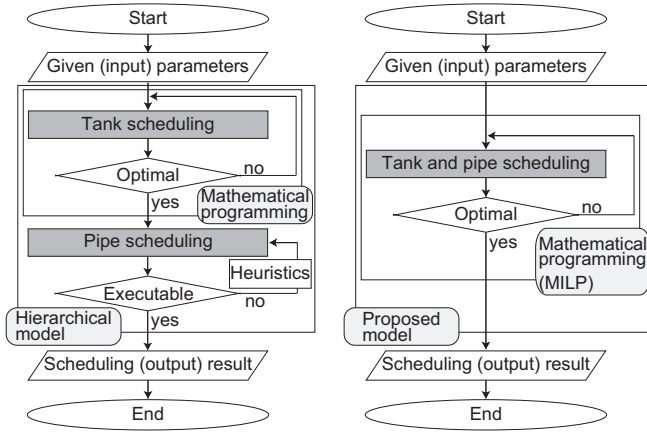
(a) Hierarchical scheduling model with heuristics

(b) Integrated scheduling model with MILP

Fig. 2.   Previous and Proposed Scheduling Approaches

(a) Paths that share the same pipe in the pipe network

(b) Paths connected from the same tank to the blender

Fig. 3.   Special Path Sets Defined with the Use of the Set Theory

after the optimal tank scheduling result was obtained as shown in Fig. 2(a). Eventually, this approach might have a case in which no executable pipe scheduling solution for the tanks is obtained. Therefore, in this paper, we propose an integrated scheduling model with the use of the MILP method considering the storage tanks and pipe network on a reciprocal basis as shown in Fig. 2(b). For this purpose, a unit-specific event-based scheduling model [13] is applied in consideration of the scheduling flexibility, number of binary variables, and strictness of constraints for the storage tanks and blender. After that, the model is extended in order to model and formulate the pipe network and constraints with the use of the set theory.

## III. SCHEDULING FOR GASOLINE BLENDING

A. Scheduling Problem

The scheduling problem for gasoline blending is to obtain the assignable, i.e., start and end timings of the component tanks, pipes, blenders, and product stock tanks while satisfying constraints and minimizing or maximizing an objective function for given parameters, such as production and delivery orders, specifications of storage tanks and blenders, storable component and product, initial amount of component and product stocks, recipe, scheduling horizon, and path list (see IV-A).

With regard to the off-site system, we assume that one blender is deployed for the mixing of gasoline components. In addition, the blender is connected to each product stock tank via one independent pipe, respectively.

B. Challenge

For the integrated gasoline-blending scheduling considering the storage tanks and pipes, it is necessary to develop a model in which (a) the consistent relation between the tanks and pipes and (b) the inflow of gasoline components into the pipes themselves are considered. Therefore, we have to tackle the following challenge:
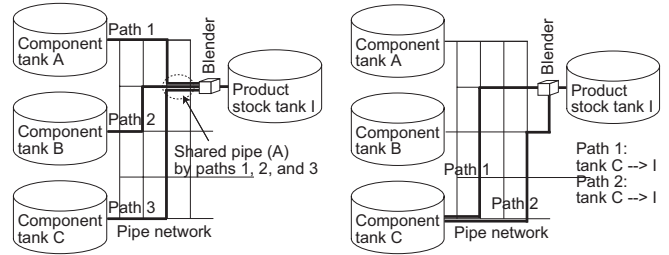
- Formulation of constraints for the structurally-complex pipe network.

Moreover, since this is a combinatorial scheduling problem which consists of the tanks, blender, and pipes, in formulating the pipe constraints, the increase of the computational cost must be restrained to complete the calculation for the scheduling within limited time frame.

C. Approach

In this paper, we consider a tank in which a gasoline component is stored and the blender as a start and goal respectively, and define the component transfer route which consists of pipes that connect these start and goal equipments as a "path." Herewith, it is possible to define the pipe network as a set of a number of the path elements. Therefore, the pipe scheduling is to assign a path from the path set to transfer the gasoline component. Furthermore, we define a set of the all assignable path elements as the whole path set. Consequently, it is possible to represent a subset of the whole path set as a special path set. From these definitions, the constraints for the structurally-complex pipe network are formulated as the MILP problem with the use of the set theory.

As for the computational cost, the number of binary variables which are used for the tank scheduling is decreased by introducing binary variables for the pipe scheduling. Moreover, in solving the MILP problem using the Branch And Bound (BAB) method, in order to restrain the increase of the number of leaves at the bottom of the BAB tree, the constrains on the pipes are formulated with the Special Ordered Set of type One (SOS1, e.g., [14] and [15]).

IV. INTEGRATED SCHEDULING MODEL CONSIDERING STORAGE TANKS AND PIPES

A. Path Set Definition with the Use of the Set Theory

Fig. 3 denotes special path sets which are defined for the pipe scheduling. A set of paths that share a pipe and a set of paths that are connected from same component tank C (start) to the blender (goal) are shown in Fig. 3(a) and Fig. 3(b), respectively. In this paper, a synopsis of elements regarding the whole path set and the special path sets described above is named "path list."
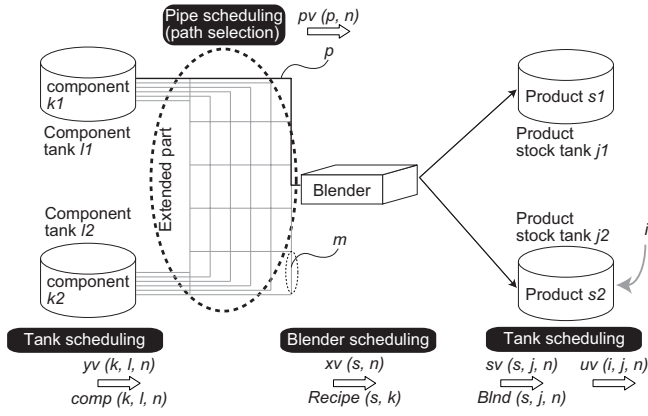
Fig. 4.   Tank, Blender, and Pipe Scheduling Constraints

B. Modeling and Formulation Using the MILP Method

As shown in Fig. 4, in this paper, a previous scheduling model [13] is extended in order to formulate the constraints on the pipes (which is a circled area with the dashed line) using the MILP method in addition to the storage tanks and blender.

The material balance constraint among the storage tanks and blender in Fig. 4 is considered by (1). Constraint (1) expresses that the required amount of component k in order to produce product s at event point n should be equal to the total amount of component k being transferred from all of the component tanks at that event point.

$$\sum_{s}[Recipe(s,k) \times \sum_{j \in J_s} Blnd(s,j,n)] =$$
$$\sum_{l \in L_k} comp(k,l,n), \forall s \in S, k \in K, n \in N \quad (1)$$

1) Indices, sets, parameter, and variables:

- **Indices**
  i = orders, j = product stock tanks, s = products, k = components, l = component tanks, n = event pints, p = paths, and m = pipes.
- **Sets**
  I = orders, J = product stock tanks, J_s = product stock tanks which can store product s, N = event points within the time horizon, S = products, K = components, K_l = components which can be stored in component stock tank l, L = component stock tanks, L_k = component stock tanks which can store component k, P = paths, $P_{sh,m}$ = paths which share pipe m, P_l = paths which are connected from component tank l to the blender, and M = pipes.
- **Parameter**
  $Recipe(s,k)$ = mix proportion of component k to product s.
- **Variables**
  $uv(i,j,n)$ = binary variables that assign the beginning of order i in tank j at event point n, $sv(s,j,n)$ = binary variables that assign product s being produced and transferred to tank j at event point n, $xv(s,n)$ =

0-1 continuous variables that assign product s being produced at event point n, $yv(k,l,n)$ = binary variables that assign component k being extracted from component tank l at event point n, $Blnd(s,j,n)$ = amount of product s being transferred from blender to tank j at event point n, $comp(k,l,n)$ = amount of component k being transferred to the blender at event pint n, and $pv(p,n)$ = binary variables that assign path p at event point n.

2) Mathematical formulation for pipe constraints:

- Coordination of the tank scheduling and pipe scheduling

This means that in a case in which gasoline component k is transferred from tank l through path p, the tank and path must be connected with each other; and vice versa, in a case in which path p is selected and used to transfer component k, tank l in which the component is stored must be connected to the path. This constraint can be formulated as (2).

$$yv(k,l,n) = \sum_{p \in P_l} pv(p,n), \forall k \in K_l, l \in L, n \in N \quad (2)$$

- Contamination prevention of gasoline components

Owing to this constraint, it is not allowed to use several paths that share the same pipe, as shown in Fig. 3(a), at the same time. Constraint (3) expresses that only one path p in path set $P_{sh,m}$ is assignable at event point n.

$$\sum_{p \in P_{sh,m}} pv(p,n) \le 1, \qquad \forall m \in M, n \in N \quad (3)$$

- Banning the use of superfluous paths

Due to this constraint, it is not allowed to use several paths connected from the same tank to the blender, as shown in Fig. 3(b), at the same time. Constraint (4) expresses that only one path p in path set P_l is assignable for the gasoline component transfer at even point n.

$$\sum_{p \in P_l} pv(p,n) \le 1, \qquad \forall l \in L, n \in N \quad (4)$$

3) Modeling techniques:

- Number of binary variables

From (2), $yv(k,l,n)$ can be replaced with $pv(p,n)$. Hence, by considering the pipe constraints, although the number of binary variables, $pv(p,n)$, is increased by $p \times n$, the number of binary variables, $yv(k,l,n)$, regarding the tank constraints is decreased by $l \times n$.

- Number of leaves at the bottom of the BAB tree

As for (3), it can be formulated as the SOS1, (6), by introducing slack variable, $slack(m,n)$, as described in (5). Here, constraint (6) expresses that only one variable equals to 1 and the others are all zero in $\{slack(m,n), pv(p,n)|\forall p \in P_{sh,m}\}$.

$$slack(m,n) + \sum_{p \in P_{sh,m}} pv(p,n) = 1,$$
$$\forall m \in M, n \in N \quad (5)$$

TABLE I
SPECIFICATIONS OF THE STORAGE TANKS

COMPONENT TANK	Component tank l						
	1	2	3	4	5	6	7
Component k that can be stored in tank l	1	2	3	4	5	1	4
Init. amount of component stock ($\times$ 100) [kl]	50						
Max. capacity ($\times$ 100) [kl]	150						
Max. flow rate ($\times$ 100) [kl/h]	30						
Min. flow rate ($\times$ 100) [kl/h]	0.3						
PRODUCT STOCK TANK	Product stock tank j		
	1	2	3
Product s that can be stored in tank j	1	2	3
Init. amount of product stock ($\times$ 100) [kl]	40	30	25
Max. capacity ($\times$ 100) [kl]	320		

TABLE II
PATHS SETS P_l AND $P_{sh,m}$

P_l	Element (path p)				
P_1	1	2	3		
P_2	4	5	6		
P_3	7	8	9		
P_4	10	11	12		
P_5	13	14	15		
P_6	16	17	18		
P_7	19	20	21		
$P_{sh,m}$	Element (path p)				
$P_{sh,1}$	2	4	10	13	
$P_{sh,2}$	1	7	11	15	19
$P_{sh,3}$	3	5	11	18	21
$P_{sh,4}$	1	5	9	16	
$P_{sh,5}$	2	6	13	17	
$P_{sh,6}$	8	9	10	15	20
$P_{sh,7}$	4	7	13	17	
$P_{sh,8}$	2	6	14	20	
$P_{sh,9}$	8	12	14	20	
$P_{sh,10}$	2	4	12	13	

TABLE III
PRODUCTION AND DELIVERY ORDERS

	Order i						
	1	2	3	4	5	6	7
Product s	1	2	1	2	2	1	3
Demand ($\times$ 100) [kl]	60	50	60	80	120	140	200
Time by which order i can start [h]	0	0	24	48	96	118	150
Time by which order i is due [h]	24	24	48	72	120	190	185
Lifting rate of order i ($\times$ 100) [kl/h]	20						

$$SOS1(m,n) =$$
$$\{slack(m,n), pv(p,n)|\forall p \in P_{sh,m}\},$$
$$\forall m \in M, n \in N \quad (6)$$

Since (4) is a similar constraint to (3), this equation can also be formulated as the SOS1. Therefore, by introducing slack variable, $slack(l,n)$, as described in (7), this equation is represented as (8).

$$slack(l,n) + \sum_{p \in P_l} pv(p,n) = 1, \quad \forall l \in L, n \in N \quad (7)$$

$$SOS1(l,n) = \{slack(l,n), pv(p,n)|\forall p \in P_l\},$$
$$\forall l \in L, n \in N \quad (8)$$

C. Objective Function

The objective function is formulated as (9). The objective of the scheduling problem is to minimize the sum of artificial variables, $arti(s,n)$, so as to determine a feasible or optimal solution with a flow rate as close to a flow rate of product s being produced and transferred to product stock tanks as possible [13].

$$minimize \sum_{s} \sum_{n} arti(s,n), \quad \forall s \in S, n \in N \quad (9)$$

V. CASE STUDY

A. Off-site System in a Petroleum Refining Plant

In the off-site system, we assume the following equipments, components, and products are arranged and treated: seven component tanks; one blender; three product stock tanks; five gasoline components; and three products (one regular and two high-octane gasolines).

1) Equipments: Specifications of the component tank and product stock tank are described in Table. I. As for the blender, the flow rate of a product being produced and transferred to product stock tanks, that is, the mixing rate is 1,000 [kl/h].

2) Path list: In the pipe network, we assume that there are three paths from each component tank to the blender. Thus, since there are seven component tanks, the following $7 \times 3 = 21$ paths are defined as the whole path set P.

- $P = \{p|p = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$

Elements (paths p) in path sets P_l and $P_{sh,m}$ are described in Table II. In this case study, the elements of $P_{sh,m}$ are generated in random manner. Additionally, $m = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, in other words, 10 pipes in set M are assumed to be shared by paths p in set $P_{sh,m}$.

3) Orders and recipe: In Table III, gasoline production and delivery orders, such as product gasoline s for order i, amount demanded, time by which order i can start and is due, and lifting rate of order i in an hour, are mentioned. In Table IV, a mix proportion of gasoline component k to produce product gasoline s is listed.

B. Numerical Experiment for Gasoline-Blending Scheduling

1) Scheduling horizon and event point n: In this experiment, the scheduling horizon is eight days, i.e., 192 hours. The number of event points, N, for the gasoline-blending scheduling is seven in consideration of the number of orders $I (= 7)$. Through a hearing investigation, we found that it takes about 8 hours for the gasoline-blending scheduling horizon (192 hours) in an actual petroleum refining plant. Hence, 8 hours are given for the scheduling time.

2) Computation environment: As for the computational performance, CPU is AMD Athlon$^{\text{TM}}$ 64×2 Dual Core Processor 3800+ 2.0 GHz and RAM is 958 MB. As an MILP solver, lp_solve is used in MATLAB 7.5.0. In this free solver, the BAB method is applied.
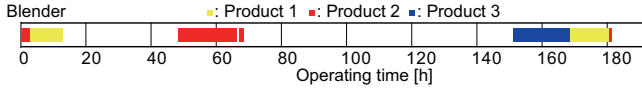
C. Gasoline-blending Scheduling Result with Two Models

Fig. 5 shows a result (gantt chart) of the gasoline-blending scheduling for the production and delivery orders considering

TABLE IV

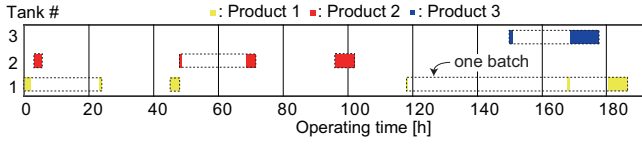| Product s	Component k (Mix proportion [%])				
	1	2	3	4	5
1	20	40	30	0	10
2	40	0	10	50	0
3	5	30	0	25	40
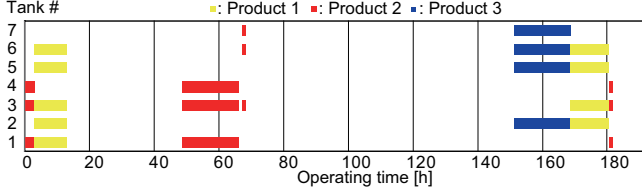
(a) Component tank l: $yv(k, l, n)$ (to the blender)

(b) Blender: $xv(s, n)$ (to product stock tank j: $sv(s, j, n)$)

(c) Product stock tank j: $uv(i, j, n)$ for a delivery

Fig. 5.   Scheduling Result with the Use of the Previous Model

(a) Component tank l: $yv(k, l, n)$ (to path p: $pv(p, n)$)

(b) Path p: $pv(p, n)$ (to the blender)
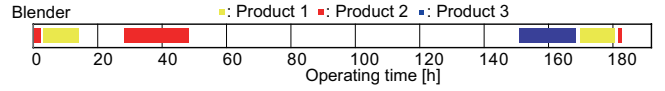
(c) Blender: $xv(s, n)$ (to product stock tank j: $sv(s, j, n)$)
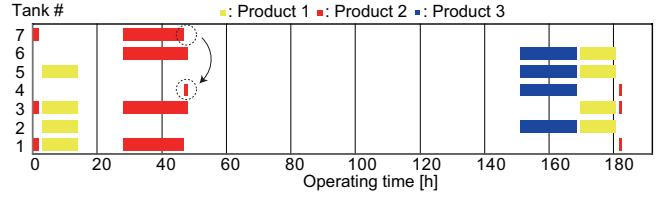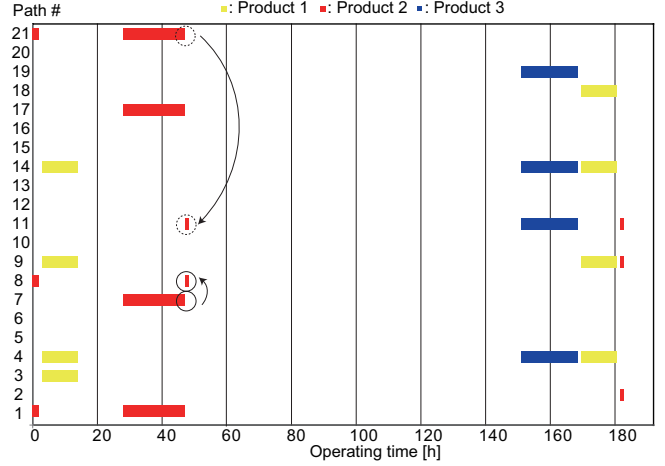

(d) Product stock tank j: $uv(i, j, n)$ for a delivery

Fig. 6.   Scheduling Result with the Use of the Proposed Model

only the storage tanks and blender with the use of the previous scheduling model. In Fig. 5(a) and Fig. 5(b), the assignment timings to the component tanks and blender for the gasoline production are shown. Fig. 5(c) shows the assignment timing to the product stock tanks for the product gasoline delivery. On the other hand, Fig. 6 shows the scheduling results considering the pipes in addition to the storage tanks and blender with the use of the proposed model. In Fig. 6(b), the assignment timing to the paths for the gasoline components transfer is shown.

From the results of Fig. 5(a) and Fig. 5(b), the gasoline production was not executed during $70 \sim 150$ [h]. The reason of this result is that (I) several scheduling solutions that minimized the objective function $arti(s, n)$ were obtained because the amount demanded of each order given in this case study was small and (II) the scheduling result, when the equalities of the constraints were true, was obtained as an optimal solution. As shown in Fig. 5(c), although the total number of delivery orders I is seven, the total number of product deliveries was 12. This is because of the following constraint in the previous model [13]: for an order, around a product stock tank, it is not allowed to transfer and store a product gasoline to the tank from
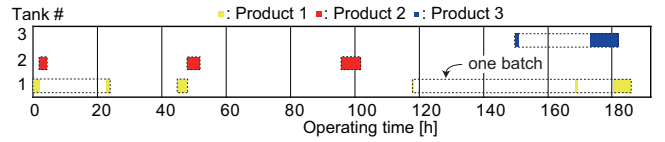
the blender and to deliver the product from the tank at the same time. Consequently, the gasoline production and the product delivery were intermittently executed during gasoline blending; and therefore, one-time product delivery was split up into several batches as depicted by dashed rectangles.

As can be seen in Fig. 6(a) $\sim$ Fig. 6(c), the gasoline production was not executed during $50 \sim 150$ [h] for similar reasons to the results in Fig. 5(a) and Fig. 5(b). Dashed circles in Fig. 6(a) show that the assigned component tank was changed from 7 to 4 during the gasoline production that had been begun since 28 [h] to produce product gasoline 2. This change is due to the assumption that gasoline component 4

TABLE V
COMPARISON OF COMPUTATIONAL COST

Two techniques are:	binary variables	# of leaves	calculation time [s]
introduced	273	12^{49}	5028
not introduced	322	24^{49}	–

is stored in component tanks 4 and 7, respectively. Dashed circles in Fig. 6(b) correspond to the ones in Fig. 6(a). The assigned path was changed from p 21 which belongs to P_7 to p 11 which belongs to P_4 when the component tank was changed from 7 to 4. This indicates that the paths were adequately selected and used depending on the change of the component tanks. Following the result, since it is not allowed to use paths 7 and 11 at the same time as mentioned in Table II, path p 7 was changed to 8 as can be depicted by the solid circles in Fig. 6(b). From Fig. 6(d), the total number of product deliveries was 11.

D. Effectiveness of the Integrated Scheduling Model

In order to evaluate a feasibility of the proposed and integrated scheduling model (see Fig. 2(b)) compared to the previous and hierarchical scheduling approach shown in Fig. 2(a), we examined whether the previous approach is able to solve the same problem given in this case study. As a result, neither a feasible nor optimal scheduling solution for the pipe network was obtained with the optimal scheduling solution regarding the storage tanks and blender. This result indicates that there was no path that satisfied the pipe constraints and it is not always true that an optimal scheduling result, considering the storage tanks and blender only, is executable. Furthermore, the objective values obtained with the both scheduling models were almost the same in the first feasible solution (22.5 and 23.0) and were the same in the optimal solution (0 and 0). This indicates that the solutions of the same quality were obtained by using the proposed scheduling model even if the pipes were additionally considered in the scheduling.

A computational cost of the scheduling for gasoline blending with the use of the proposed model is described in Table V. In the table, the upper row shows the results obtained by introducing two modeling techniques, and the lower row shows the results obtained without using the modeling techniques.

As for the number of binary variables, although it was normally 322 $(175 + 147)$ in total due to $pv(p, n)$ introduced to consider the 168 $(1255 - 1087)$ pipe constraints, 49 variables of $yv(k, l, n)$ were canceled as described in IV-B.3; and therefore, it was 273 in total. Moreover, the number of leaves was reduced to 12^{49} from 24^{49} by formulating the pipe constraints with the SOS1. In this result, the optimal solution was obtained within 1.4 [h] (5028 [s]). From the viewpoint of the calculation time for the scheduling given in V-B.1, the proposed scheduling model is reasonable. On the other hand, we could not obtain the optimal solution in case of equations, (3) and (4), were used without applying the SOS1 for the pipe constraints formulation even if the

calculation was conducted for 48 hours.

From these results, the effectiveness of the integrated scheduling model with the use of two modeling techniques considering the pipe network in addition to the storage tanks and blender is finally shown.

VI. CONCLUSION

In this paper, a novel scheduling model with the MILP method for gasoline blending in consideration of storage tanks and a pipe network was proposed. For the modeling and formulation of the pipe network in addition to the storage tanks and blender, a previous scheduling model was extended, and additional constraints were provided with the use of the set theory. In a case study, previous and proposed scheduling models were compared and discussed in terms of the scheduling feasibility and the quality of the solution. Moreover, the computational cost of the proposed scheduling model with and without two modeling techniques were compared and discussed. From these results, the effectiveness of the proposed and integrated scheduling model was shown.

REFERENCES

[1] E. Kondili et at., General Algorithm for Short-term Scheduling of Batch Operations - I. MILP Formulation, *Computers & Chemical Engineering*, vol. 17, 1993, pp 211–227.

[2] C.C. Pantelides, Unified Framework for Optimal Process Planning and Scheduling, *Proc. of the 2nd Int. Conf. on Foundations of Computer-aided Process Operations*, 1994, pp. 253–274.

[3] M.R. Almeida et at., Optimizing the Production Scheduling of a Petroleum Refinery Through Genetic Algorithms, *Int. Journal of Ind. Eng.: Theory, Applications and Practice*, vol. 10, 2003, pp. 35–44.

[4] W.H.M. Raaymakers et at., Scheduling Multipurpose Batch Process Industries with No-Wait Restrictions by Simulated Annealing, *European Journal of Operational Research*, 126, 2000, pp. 131–151.

[5] A.-P. Catherine et at., A Two-stage Methodology for Short-term Batch Plant Scheduling: Discrete-event Simulation and Genetic Algorithm, *Computers & Chemical Engineering*, vol. 22, 1998, pp. 1461–1481.

[6] J.M. Pinto et at., A Continuous Time Mixed Integer Linear Programming Model for Short Term Scheduling of Multistage Batch Plants, *Ind. Eng. Chem. Res.*, vol. 34, 1996, pp. 3037–3051.

[7] M. G. Ierapetritou et at., Effective Continuous-Time Formulation for Short-Term Scheduling: 1. Multipurpose Batch Processes, *Ind. Eng. Chem. Res.*, vol. 37, 1998, pp. 4341–4359.

[8] M. G. Ierapetritou et at., Effective Continuous-Time Formulation for Short-Term Scheduling: 2. Continuous and Semi-continuous Processes, *Ind. Eng. Chem. Res.*, vol. 37, 1998, pp. 4360–4374.

[9] M. G. Ierapetritou et at., Effective Continuous-Time Formulation for Short-Term Scheduling: 3. Multiple Intermediate Due Dates, *Ind. Eng. Chem. Res.*, vol. 38, 1999, pp. 3446–3461.

[10] K. Glismann et at., Short-Term Scheduling and Recipe Optimization of Blending Processes, *Computers & Chemical Engineering*, vol. 25, 2001, pp. 627–634.

[11] M. Djukanovic et at., Fuzzy Linear Programming Based Optimal Fuel Scheduling Incorporating Blending/Transloading Facilities, *IEEE Transaction on Power Systems*, vol. 11, 1996, pp. 1017–1023.

[12] C.A. Méndez et at., A Simultaneous Optimization Approach for Off-line Blending and Scheduling of Oil-refinery Operations, *Computers & Chemical Engineering*, vol. 30, 2006, pp. 614–634.

[13] Z. Jia et at., Mixed-Integer Linear Programming Model for Gasoline Blending and Distribution Scheduling, *Ind. Eng. Chem. Res.*, vol. 42, 2003, pp. 825–835.

[14] E.M.L. Beale et at., Special Facilities in a General Mathematical Programming System for Non-convex Problems Using Ordered Sets of Variables, *Proceedings of the Fifth International Conference on Operational Research*, 1970. pp. 447–454.

[15] E.M.L. Beale et at., Global Optimization Using Special Ordered Sets, *Mathematical Programming*, vol. 10, 1976, pp. 52–69.